Properties of DFT

1. If x(n) and X(k) are an N-point DFT pair, then x(n+N)=x(n)
   a) True b) False

   Answer: a
   Explanation: We know that the expression for an DFT is given as

   $$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

   Now take x(n)=x(n+N) ⇒

   $$X_1(K) = \sum_{n=0}^{N-1} x(n+N) e^{\frac{-j2\pi kn}{N}}$$

   Let n+N=*l*

   $$X_1(K) = \sum_{n=0}^{N-1} x(l) e^{\frac{-j2\pi kl}{N}} = X(K)$$

2. If x(n) and X(k) are an N-point DFT pair, then X(k+N)=?

   a) X(-k) b) -X(k) c) X(k) d) None of the mentioned

   Answer: c
   Explanation: We know that

   $$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K) e^{\frac{j2\pi kn}{N}}$$

   $$let\ X(K) = X(k+N) \Rightarrow$$

   $$x_1(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K+N) e^{\frac{j2\pi kn}{N}} = x(n)$$

   Therefore, we have X(k)=X(k+N)

3. If X1(k) and X2(k) are the N-point DFTs of x1(n) and x2(n) respectively, then what is the N-point DFT of x(n)=ax1(n)+bx2(n)?
   a) X1(ak)+X2(bk) b) aX1(k)+bX2(k)
   c) e^{ak}X1(k)+e^{bk}X2(k) d) None of the mentioned

   Answer: b
   Explanation: We know that, the DFT of a signal x(n) is given by the expression

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$$\boldsymbol{X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}}$$

Given x (n) =ax$_1$ (n) +bx$_2$ (2)

$$X(K) = \sum_{n=0}^{N-1} ax_1(n) + bx_2(2) e^{\frac{-j2\pi kn}{N}}$$

$$X(K) = \sum_{n=0}^{N-1} ax_1(n) e^{\frac{-j2\pi kn}{N}} + \sum_{n=0}^{N-1} bx_2(2) e^{\frac{-j2\pi kn}{N}}$$

$$X(K) = a \sum_{n=0}^{N-1} x_1(n) e^{\frac{-j2\pi kn}{N}} + b \sum_{n=0}^{N-1} x_2(2) e^{\frac{-j2\pi kn}{N}}$$

$$\boldsymbol{X(K) = aX_1(K) + bX_2(K)}$$

4. If x(n) is a complex valued sequence given by x(n)=xR(n)+jxI(n), then what is the DFT of x_R(n)?

$$\boldsymbol{a)\ \sum_{n=0}^{N} x_R(n) \cos\frac{2\pi kn}{N} + x_I(n) \sin\frac{2\pi kn}{N}}$$

$$\boldsymbol{b)\ \sum_{n=0}^{N} x_R(n) \cos\frac{2\pi kn}{N} - x_I(n) \sin\frac{2\pi kn}{N}}$$

$$\boldsymbol{c)\ \sum_{n=0}^{N-1} x_R(n) \cos\frac{2\pi kn}{N} - (n) \sin\frac{2\pi kn}{N}}$$

$$\boldsymbol{d)\ \sum_{n=0}^{N-1} x_R(n) \cos\frac{2\pi kn}{N} + x_I(n) \sin\frac{2\pi kn}{N}}$$

Answer: d

Explanation: Given x(n)=x_R(n)+jx_I(n)=> x_R(n)=1/2(x(n)+x*(n))

Substitute the above equation in the DFT expression

Thus we get,

$$\boldsymbol{\sum_{n=0}^{N-1} x_R(n) \cos\frac{2\pi kn}{N} + x_I(n) \sin\frac{2\pi kn}{N}}$$

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5. If x(n) is a real sequence and X(k) is its N-point DFT, then which of the following is true?

   a) X(N-k)=X(-k) b) X(N-k)=X*(k)

   c) X(-k)=X*(k) d) All of the mentioned

   Answer: d

   Explanation: We know that

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

   Now

$$X(N-K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi (N-K)n}{N}}$$

$$X^*(K) = X(-k)$$

   Therefore,

   X(N-k)=X*(k)=X(-k)

6. If x(n) is real and even, then what is the DFT of x(n)?

   $\boldsymbol{a)} \sum_{n=0}^{N-1} \boldsymbol{x(n)\sin\frac{2\pi kn}{N}}$ $\boldsymbol{b)} \sum_{n=0}^{N-1} \boldsymbol{x(n)\cos\frac{2\pi kn}{N}}$

   $\boldsymbol{c)} \boldsymbol{-j} \sum_{n=0}^{N-1} \boldsymbol{x(n)\sin\frac{2\pi kn}{N}}$ $\boldsymbol{d)}$None of the mentioned

   Answer: b

   Explanation: Given x(n) is real and even, that is x(n)=x(N-n)

   We know that $X_I(k)=0$. Hence the DFT reduces to

$$\boldsymbol{X(K) = \sum_{n=0}^{N-1} x(n)\cos\frac{2\pi kn}{N}}$$

7. If x(n) is real and odd, then what is the IDFT of the given sequence?

   $\boldsymbol{a)} \boldsymbol{j\frac{1}{N}} \sum_{k=0}^{N-1} \boldsymbol{X(K)\sin\frac{2\pi kn}{N}}$ $\boldsymbol{b)} \boldsymbol{\frac{1}{N}} \sum_{k=0}^{N-1} \boldsymbol{X(K)\cos\frac{2\pi kn}{N}}$

   $\boldsymbol{c)} \boldsymbol{-j\frac{1}{N}} \sum_{k=0}^{N-1} \boldsymbol{X(K)\sin\frac{2\pi kn}{N}}$ $\boldsymbol{d)}$None of the mentioned

Answer: a

Explanation: If x(n) is real and odd, that is x(n)=-x(N-n), then XR(k)=0. Hence X(k) is purely imaginary and odd. Since XR(k) reduces to zero, the IDFT reduces to

$$x\,n = j\frac{1}{N}\sum_{k=0}^{N-1} X(K) sin\frac{2\pi kn}{N}$$

8. If x1(n),x2(n) and x3(m) are three sequences each of length N whose DFTs are given as X1(k),X2(k) and X3(k) respectively and X3(k)=X1(k).X2(k), then what is the expression for x3(m)?

   a) $\sum_{n=0}^{N-1} x_1\ n\ x_2\ m+n$

   b) $\sum_{n=0}^{N-1} x_1\, n\, x_2(m-n)$

   c) $\sum_{n=0}^{N-1} x_1\ n\ x_2(\ m-n\ )_N$

   d) $\sum_{n=0}^{N-1} x_1\, n\, x_2((m+n))_N$

Answer: c

Explanation: If x1(n),x2(n) and x3(m) are three sequences each of length N whose DFTs are given as X1(k),X2(k) and X3(k) respectively and X3(k)=X1(k).X2(k), then according to the multiplication property of DFT we have x3(m) is the circular convolution of x1(n) and x2(n).

9. What is the circular convolution of the sequences x1(n)={2,1,2,1} and x2(n)={1,2,3,4}?

   a) {14,14,16,16} b) {16,16,14,14} c) {2,3,6,4} d) {14,16,14,16}

Answer: d

Explanation: We know that the circular convolution of two sequences is given by the expression

$$x\,m = \sum_{n=0}^{N-1} x_1\, n\, x_2(\ m-n\ )_N$$

For m=0,x2((-n))4={1,4,3,2}
For m=1,x2((1-n))4={2,1,4,3}
For m=2,x2((2-n))4={3,2,1,4}
For m=3,x2((3-n))4={4,3,2,1}
Now we get x(m)={14,16,14,16}.

10. What is the circular convolution of the sequences x1(n)={2,1,2,1} and x2(n)={1,2,3,4}, find using the DFT and IDFT concepts?

   a) {16,16,14,14} b) {14,16,14,16} c) {14,14,16,16} d) None of the these

Answer: b

Explanation: Given x1(n)={2,1,2,1}=>X1(k)=[6,0,2,0] Given x2(n)={1,2,3,4}=>X2(k)=[10,-2+j2,-2,-2-j2] when we multiply both DFTs we obtain the product X(k)=X1(k).X2(k)=[60,0,-4,0] By applying the IDFT to the above sequence, we get **x(n)={14,16,14,16}.**

11. If X(k) is the N-point DFT of a sequence x(n), then circular time shift property is that N-point DFT of $x((n-l))_N$ is $X(k)e^{-j2\pi kl/N}$.
    a) True
    b) False

    Answer: a

    Explanation: According to the circular time shift property of a sequence, If X(k) is the N-point DFT of a sequence x(n), then the N-pint DFT of $x((n-l))_N$ is $X(k)e^{-j2\pi kl/N}$.

12. If X(k) is the N-point DFT of a sequence x(n), then what is the DFT of x*(n)?

    a) X(N-k) b) X*(k) c) X*(N-k) d) None of the mentioned

    Answer:c

    According to the complex conjugate property of DFT, we have if X(k) is the N-point DFT of a sequence x(n), then what is the DFT of x*(n) is X*(N-k).